

Dealing with indeterminacy in spatial descriptions

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Abstract How do people tackle indeterminate spatial descriptions, that is those descriptions for which several representations are possible? Take for instance the two following statements: *B is to the left of A*, *C is to the left of A*. This description is indeterminate because it is compatible with at least two possibilities: (1) C B A; (2) B C A. Studies on human reasoning have shown that people tend to reduce the complexity of such indeterminate descriptions by representing only one possibility. Which one do people favour? Is one possibility easier to work out than the other? Is one possibility more plausible than the other? Two competing hypotheses make different predictions about the representation people favour. If the building of the representation is driven by what we call *manipulation difficulty*, then (1) is more likely to be constructed than (2) because (2) results from reorganising the representation following the first statement where B is adjacent to A (i.e. B A) while (1) is just an extension of this initial representation. However, if the representation process is driven by *pragmatic factors*, then (2) is more likely to be built than (1) because the second statement could be interpreted as implicating “C is not to the left of B”. Indeed, if C had

been to the left of B it would have been more appropriate to utter, “C is to the left of B” rather than “C is to the left of A”. Data from several experiments show that both manipulation difficulty and pragmatic factors play a role in determining participants’ representations.

Introduction

Imagine that five good friends spending the holidays together are having a discussion in a tent about what to do for the evening. Here is how they are seated:

(A) George is to the left of Nicolas
Romano is to the right of Nicolas
Tony is in front of George
Angela is in front of Romano

How will you process such a description? You may think that rather than representing each statement separately it would be better to work out a single and unified representation that integrates all the information contained in the description in a form of a spatial array:

George	Nicolas	Romano
Tony		Angela

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Once the representation is achieved it will enable you to identify new links that were not explicitly stated in the description such as the relation between George and Romano (i.e. George is to the left of Romano) or the relation between Tony and Angela (i.e. Tony is to the left

of Angela). Numerous cognitive scientists have argued that people do represent spatial relational descriptions, and more generally speaking relational descriptions displaying comparative expressions (i.e. better than, taller than, etc.), by building this type of representation (De Soto, London, & Handel, 1965; Huttenlocher, 1968; Potts, 1972; Byrne & Johnson-Laird, 1989) also called *mental models* (Johnson-Laird, 1983). A mental model is typically defined as an analogical representation whose structure is isomorphic to the structure of the situation it describes (Johnson-Laird, 1983).

The idea that people represent relational descriptions with mental models has received support from various types of findings in the relational reasoning literature. For instance, Vandierendonck and De Vooght (1997) have investigated how working memory is involved in the representation of spatial and temporal reasoning problems by means of a dual-task interference paradigm (see also Klauer, Stegmaier, & Meiser, 1997). Their data tend to show that the representation of relational descriptions is made in a visuo-spatial format, a claim that fits with the idea that the nature of mental models is analogical. In their experiments, participants had to tackle a relational reasoning task and a secondary interfering task. When the secondary task interfered with spatial rehearsal, the results indicate that problem accuracy was impaired and that the reading of the premises was enhanced. However, when the secondary task interfered with linguistic-phonological processes, the reading time of the premises did not increase (Experiment 2). According to Vandierendonck and De Vooght, this shows that the visuo-spatial working memory subsystem, but not the phonological subsystem, is engaged in the representation phase of the premises. In the same vein, Goel and Dolan (2001) carried out an fMRI study and observed that a bilateral occipital–parietal–frontal network typically involved in visuo-spatial manipulation tasks was activated with three-term relational reasoning problems. They interpret this result as an indication that people resolve such problems by relying on spatial strategies (see also, Knauff, Mulack, Kassubek, Salih, & Greenlee, 2002).

However, the data most often cited in support of the mental model approach come from studies analysing reasoning performance as a function of the number of models. The core prediction of Mental Model Theory is that problem difficulty increases with the number of models. The following reasoning problem which is compatible with two models is therefore more difficult than a single-model problem (Byrne & Johnson-Laird, 1989):

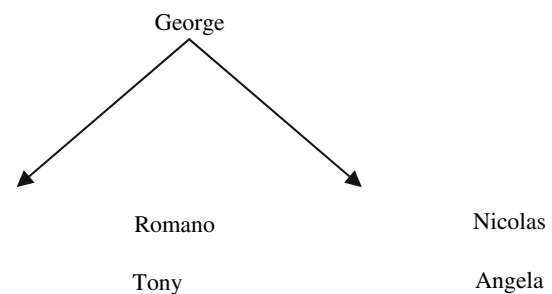
- (B) George is to the left of Nicolas
 Romano is to the left of Nicolas
 Tony is in front of Romano
 Angela is in front of Nicolas

What is the relation between Tony and Angela?

Model 1:			Model 2:		
George	Romano	Nicolas	Romano	George	Nicolas
	Tony	Angela	Tony	Angela	

This problem gives rise to two mental models because the first two premises convey an indeterminacy regarding the relation between George and Romano (George could be to the left or to the right of Romano). However, the two-models are compatible with the same conclusion regarding the relation between Tony and Angela (Tony is to the left of Angela).

The mental model prediction linking problem difficulty to the number of models was initially supported by Byrne and Johnson-Laird's (1989) study and replicated by many others (Boudreau & Pigeau, 2001; Byrne & Johnson-Laird, 1989; Carreiras & Santamaria, 1997; Roberts, 2000; Schaeken, Giroto, & Johnson-Laird, 1998; Schaeken & Johnson-Laird, 2000; Schaeken, Johnson-Laird, & d'Ydewalle, 1996a, b; Vandierendonck & De Vooght, 1997, 1998). The reason why two-model problems are more difficult than single-model problems as in (A) is simply that it is harder to maintain two models in working memory than a single one. Given the limitations of working memory, participants are likely to construct only one of the possibilities compatible with a given description and neglect the other or to construct a single model that represents the two possibilities as presented in the following model (Vandierendonck, Dierckx, & De Vooght, 2004; Schaeken, Van der Henst, & Schroyens, 2007):



Although many studies aimed at ascertaining that mental models were involved in the representation of spatial descriptions and that two-model problems were harder than single-model problems, few of them consider the factors that favour the construction of one mental model over another. When only one possibility is represented, as it is often claimed to be the case for two-model problems, which one is constructed and which one is left out? Is one mental model easier to construct than the other? Does one model provide a better representation of what is conveyed by the premises than the other?

We aim to address these questions by investigating indeterminate relational descriptions that can typically be represented by several models. We think that two types of factors may contribute to the construction of the model that will ultimately be constructed. On the one hand, given the limitation of our cognitive skills one may expect participants to construct the less demanding model. On the other hand, given that the purpose of our cognitive system is also to come up with a faithful representation of the world, one may expect that people could build a less economical model if it provides a more appropriate representation. Applied to indeterminate spatial descriptions, we believe that these two constraints should prompt the construction of two distinct kinds of models.

Let us spell out how these constraints may determine the representation process by analysing the two following statements:

- (1) George is to the left of Nicolas
- (2) Tony is to the left of Nicolas

Given the indeterminacy of the relation between George and Tony, there are at least two models compatible with those statements:

Model 1			Model 2		
Tony	George	Nicolas	George	Tony	Nicolas

First, consider the hypothesis that model construction is driven by a principle of economy, a view that is largely in line with the mental model approach advocated by Johnson-Laird and his colleagues.¹ According to this hypothesis, participants should favour the less demanding model over the more demanding one. In other words, participants should choose the representation which follows from the easiest manipulation integrating the second premise to the existing model of the first premise. Here is a more precise decomposition of this process. When a participant represents the first statement, she should come up with the following initial model:

George	Nicolas
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¹ One of the hallmarks that provides Mental Model Theory with an “economical” dimension is the principle of truth. This principle guides the construction of mental models in propositional reasoning. It stipulates that reasoners tend to reduce the load on working memory by constructing mental models that only represent what is true but not what is false.

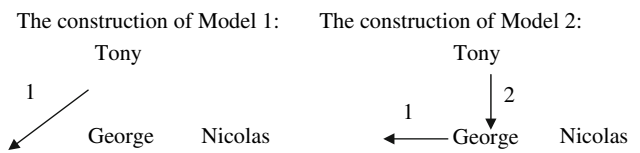


Fig. 1 Two types of model construction

The next incoming piece of information will then be integrated with this initial part. The difficulty one encounters when integrating a new item in the initial model can be linked to the degree to which this initial model has to be modified in order to achieve this integration. The more modifications required of the initial representation, the more demanding the integration. In order to construct Model 2 and to insert Tony between George and Nicolas, the initial model has to be modified in such a way that the “George” item will be moved to the left (see Fig. 1). In contrast, the construction of Model 1 is the result of the simple addition of a new item to the initial part, which remains unchanged otherwise since Tony is put to the left of George (see Fig. 1). Consequently, if the model construction is mainly guided by what we refer to as *manipulation difficulty*, Model 1 should be favoured over Model 2.

The view that the construction of spatial mental models follows a principle of economy has been previously advocated by Rauh and his colleagues (Rauh, 2000; Rauh et al., 2005). These researchers have investigated the mental models that are preferentially built by participants when they reason on the basis of less natural but also less ambiguous spatial expressions involving interval relations (see also Knauff, Rauh, & Schlieder, 1995). Here are three examples of the 13 relations they used: *Interval X contains Interval Y left-justified*; *Interval X lies right-justified of Interval Y*; *Interval X overlaps Interval Y from the left*. Using reasoning tasks, they observed that the participants’ preferred solutions were in line with (three) principles of representational economy (Rauh et al., 2005, p. 249).²

We now turn to a second hypothesis which is based on a pragmatic analysis of the task. According to the pragmatic approach, the spatial description provided in statements (1) and (2) can be considered as an online description made by a speaker to a hearer. The hearer can process each statement as if it were the most appropriate way for the speaker to express what she is observing. This is why we will focus on spatial prepositions that were uttered under more natural conditions, so as to investigate pragmatic effects.

² A linearization principle (“preferred solutions follow a linear order of start points and end points”); a regularization principle (“mental configurations of intervals incorporate point incidences in only those cases where they are unavoidable”) and a unification principle (“in the case of using the inverse of the first-premise-relation in the second premise, people prefer to equalize the end terms”).

Let us now figure out how a speaker, whose purpose is to be helpful to the hearer, is likely to describe a spatial configuration by using the expression “to the left of” or “to the right of”. Imagine that the initial spatial configuration seen by the speaker is:

George	Nicolas
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She could describe this scene very straightforwardly by saying that “George is to the left of Nicolas” (equally as “Nicolas is to the right of George”). Let us now assume that a new item (i.e. Tony) arrives and that the configuration seen by the speaker is the following:

Tony	George	Nicolas
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The speaker has two options: The first one is to say that “Tony is to the left of Nicolas” and the second one is to say that “Tony is to the left of George”. Both statements are equally true but from a communicative viewpoint, the second option is more appropriate than the first. There are two main reasons for this:

- a. The first option yields two possibilities and therefore creates an ambiguity (i.e. Tony could be to the left of Nicolas and to the right of George; or Tony could be to the left of Nicolas and also to the left of George).
- b. The first option focuses on the relation between Tony and Nicolas whereas the most immediate item to which Tony is related is George but not Nicolas.

Since the second option does not give rise to any indeterminacy and focuses on the item to which Tony is immediately related (i.e. George) it is a more helpful description of the configuration and the speaker is more likely to choose it. In other words, if the speaker states (2) in the context of (1) (repeated below for convenience), it arguably implies that “Tony is immediately to the left of Nicolas” (“Tony is not to the left of George”).

1. George is to the left of Nicolas
2. Tony is to the left of Nicolas

As just shown, if Tony had been to the left of George, it would have been more appropriate to say so. Since the speaker does not, the hearer may infer that “Tony is not to the left of George”. By uttering statement (2) after statement (1) the speaker is thus more likely to have conveyed a configuration compatible with Model 2 rather than a configuration compatible with Model 1. It follows

that if the hearer assumes the speaker to have provided the best online description, she should be more likely to construct Model 2 than Model 1.

In brief, the “manipulation difficulty” hypothesis predicts that Model 1 will be favoured over Model 2 whereas the “pragmatic” hypothesis predicts the opposite. Our experiments aim to explore the extent to which these two factors contribute to determining the representation people favour. Experiment 1 involves the standard spatial reasoning problems used in the reasoning literature and analyses the types of erroneous answers made for two-model problems. Erroneous answers are standardly assumed to be compatible with only one of the two possible models and are therefore likely to reveal the model built by the participants (Johnson-Laird & Byrne, 1991). Experiment 2 uses the same descriptions as those in Experiment 1 but involves a more direct method to investigate how participants deal with indeterminacy by asking them to draw a single possibility compatible with the description they read. Experiment 3 manipulates the pragmatic factor and Experiment 4 tests the prediction that the pragmatic representation will be favoured if manipulation difficulty is neutralised.

Experiment 1

Experiment 1 investigates the type of mental model on which participants rely when they have to answer a question about an indeterminate spatial description. According to Mental Model Theory, an erroneous answer typically results from the construction of a subset of all possible models whereas the correct answer would have required the construction of all models. An erroneous answer may thus reveal the models participants construct in order to solve the problem they have to tackle. Consider the following problem:

- (C) The lemon is to the left of the apple
 The orange is to the left of the apple
 The pear is in front of the lemon
 The nectarine is front of the orange
 What is the relation between the pear and the nectarine?

This problem is compatible with the two types of models previously described:

“3rd element-Outside” model:			“3rd element-Inside” model:		
orange	lemon	apple	lemon	orange	apple
nectarine	pear	pear	nectarine		

Note that the “3rd element Outside” model (hereafter “Outside” model) requires a minimal amount of

manipulation effort whereas the “3rd element-Inside” model (hereafter “Inside” model) requires more manipulation (reorganising the model linked to the first premise by inserting a third element) whilst being more pragmatically felicitous. If participants succeed in constructing the two models, they should be able to respond that the relation between the pear and the nectarine is indeterminate (this will be referred to as the “correct” answer). However, an answer that reflects a minimal amount of manipulation (“Outside” model) would be “the pear is to the right of the nectarine” while an answer influenced by pragmatics (“Inside” model) would be “the pear is to the left of the nectarine”. Although neither of these is correct alone, they reveal subjects’ individual preferences.

Participants

Seventeen undergraduate social sciences students from the University Lumière (Lyon) took part in this experiment. They received a book for their participation. All were native speakers of French.

Method

Material and design

Participants were presented with spatial descriptions and were required to answer a question about a spatial relation that was not explicitly given in the description (see example C above). Each description contained four sentences involving a set of five items composed of fruits or vegetables whose names started with different letters (see the example given above). The first two sentences involved the relational expressions “to the left of” (translated from the French “à gauche de”) or “to the right of” (“à droite de”) and the last two sentences involved the relational expression “in front of” (“devant”). The four sentences were followed by a question that asks about the relation between the last two items introduced in the description (i.e. the pear and the nectarine in the example given above). Participants received eight indeterminate problems compatible with two models and eight determinate problems which served as fillers. The eight indeterminate descriptions correspond to the eight different ways to combine the first two sentences in order to express the indeterminacy (see Table 1). The eight determinate descriptions correspond to the eight different ways to combine the first two sentences in order to express a determinate relation between three items. Thus, half of the descriptions involved *homogeneous* problems (i.e. the first two sentences contained the same spatial relations) and the other

half involved *heterogeneous* problems (i.e. the first two sentences contained different spatial relations). Problems were provided in a booklet (one page per problem) and were arranged in a random order. An example involving a four-item determinate problem (with three premises) was provided with its correct solution on the instruction page.

Procedure

The premises and the question were presented together on the same page and participants had to write their answer below the question. They received the following instructions: “In this experiment, you will first have to read descriptions involving several sentences each containing a relational expression such as “to the left of”, “to the right of” or “in front of”. Then, for each description you will have to answer a question concerning the relation between two items. To answer the question, you are not allowed to draw the relations expressed in the description or to take notes”.³ Participants were tested individually and acted as their own control.

Results and discussion

Our data are in line with previous findings on two counts. First, the rate of correct responses are in line with the Mental Model Theory since multiple-model problems yielded significantly fewer correct answers than single-model problems (13% vs. 79% Wilcoxon signed rank test, $N = 17$, $z = 3.52$, $P < 0.001$). Although the difference is in the same direction as the one reported in the literature, the rates of correct performance for multiple-model problems having no valid conclusion are relatively low in comparison with other studies (the percentage of correct answers

³ The experimenter did not provide any indication about the way to solve indeterminate problems since this may have helped participants find the correct answer to such problems in the test phase (i.e. when the relation posed in the question is indeterminate) and may reduce the occurrence of wrong answers. As observed by Schaeken and Van der Henst (2005), informing participants that the answer “Nothing follows” (i.e. the relation is indeterminate) is correct for some problems leads to relatively high rates of correct performance for indeterminate problems. This arguably results from a strategy consisting in (a) looking for indeterminacy in the first place and in (b) not engaging fully in the construction of mental models. In this experiment, we were more interested in erroneous answers that reveal which model is built (and which one is neglected) than correct answers per se showing that indeterminacy was successfully identified. We did not provide any clue that may encourage the detection of indeterminacy. Hence, participants were not informed that some problems were indeterminate.

Table 1 The percentages of answer type for each problem

	Problem type								Mean
	A left B A left C	A left B C right A	A right B A right C	A right B C left A	B left A A right C	B left A C left A	B right A A left C	B right A C right A	
Correct	17.6	11.8	5.9	11.8	11.8	17.6	11.8	17.6	13.2
Outside	29.4	23.5	17.6	29.4	17.6	17.6	35.3	47.1	27.2
Inside	29.4	41.2	41.2	29.4	47.1	35.3	35.3	17.6	34.6
Other errors	23.5	23.5	35.3	29.4	23.5	29.4	17.6	17.6	25
Total	100	100	100	100	100	100	100	100	100

Only the first two sentences of the problems are presented in the table since the last two sentences were identical across problems

usually ranges from 40 to 60%.⁴ We presume that this is due to the fact that participants receive no specific instructions regarding the possibility of answering “indeterminate”. In prior studies, the experimenter more or less explicitly mentions the possibility of such an answer. In most experiments participants are told that if they think there is no definite answer they should say so in their answer. Even more explicitly, Roberts (2000, Experiment 3) informed participants that some descriptions were ambiguous and that, in these cases, the question had no valid answer. As discussed in the procedure section and in Schaeken and Van der Henst (2005), this may help participants discover indeterminacy.

Second, participants’ wording in the conclusions resemble what has been previously observed by Van der Henst and Schaeken (2005): Among all responses using “to the left of” or “to the right of”, 68% were of the first kind and 32% of the second kind. The mean difference between “Left” and “Right” answers across all participants was 4.65 (SD = 5.75), a value that significantly differs from 0 ($t(16) = 3.33$, $P < 0.005$). This preference for “left” conclusions could be explained by the influence of left-to-right reading habits on mental scanning (see Van der Henst & Schaeken, 2005).

We now turn to the analysis of “incorrect” responses in indeterminate problems. Table 1 presents the percentages of the different answer types for the eight indeterminate problems. First, a significant proportion of the answers (25%) were classified as unspecified because the spatial relation between the items mentioned in the question was not expressed with “to the left of” or “to the right of”. These include answers like “they are side by side”, “they are at the same level”, “they are at the same place”. Second, 61.8% of the answers were compatible with one of

the two models of the description. The rate of answers compatible with the “Outside model (27.2%) did not significantly differ from the rate of answers compatible with the “Inside” model (34.6%). The mean difference between “Inside” and “Outside” answers across all participants was 0.59 (SD = 2.79), a value that does not significantly differ from 0 ($t(16) = 0.87$ NS). An analysis of participants’ pattern of answers confirms this absence of difference: Five participants produced more answers compatible with the “Outside” model than with the “Inside” model, six participants showed an opposite pattern, and six participants did not show any difference. Third, the rate of “Outside” or “Inside” answers did not significantly differ across the type of indeterminate problems: For homogeneous and heterogeneous problems, 28 and 26% of the answers were compatible with the “Outside” model respectively (Wilcoxon signed rank test, $N = 17$, $T = 13$, $z = 0.17$, NS) and 31 and 38% of the answers were compatible with the “Inside” model respectively (Wilcoxon signed rank test, $N = 17$, $T = 28$, $z = 0.86$, NS).

Experiment 1 failed to exhibit a clear preference for one type of model over another. This may result from an equal influence of the two factors aforementioned (manipulation difficulty and pragmatics). Another possibility is that none of the factors had an influence suggesting that other factors we did not consider here determine the model people construct or more radically that the selection of one model over another is completely random (nondeterministic). This is why we need to know whether or not each of the factors, manipulation difficulty and pragmatics affect the way we deal with indeterminate spatial descriptions in situations other than the one investigated in Experiment 1. In the following experiments, we adopt a procedure that allows for a more direct observation of the representation participants produce (as opposed to inferring the model participants construct from the erroneous responses they provide). First, participants are invited to externalise their representation with paper and pencil or with real objects. Second, they have to represent only one of the possibilities compatible with the description they receive.

⁴ Except for Byrne and Johnson-Laird’s study which reports 18% of correct answers for the same kind of problems. However, in Byrne and Johnson-Laird’s study the premises were orally, and thus sequentially, presented. This easily explains the low level of performance in their study.

Experiment 2

In Experiment 2, participants did not have to answer a question about a description but had to produce an external representation of the description with paper and pencil.

Method

Participants

Seventeen undergraduate social sciences students from the University Louis Lumière (Lyon) took part in this experiment. They received a book for their participation. All were native speakers of French. None of them participated in Experiment 1.

Material and design

In this experiment participants were presented with the same 16 determinate and indeterminate descriptions as in the previous experiment, but instead of answering a question, they were required to draw a *schema* (in fact we used the French word *schéma*) including the items of the description. Again, the 16 descriptions were provided in a booklet (one page per problem) and were arranged in a random order. As in Experiment 1, the eight determinate descriptions served as fillers. Participants had to draw the spatial configuration of the five items. An example involving a four-item determinate description was represented by a schema that displayed the names of the four items. This example was designed to incite participants to use the names of the items in their own schemas rather than drawing the items themselves.

Procedure

The purpose of this experiment was obviously not to assess the ability to represent the two possibilities compatible

with an indeterminate description but it was rather to find out which of these two possibilities would be favoured. Consequently, participants were asked to draw only one schema even when they thought that several possibilities were compatible with the description. More specifically, the instructions read as follows: “In this experiment, you will first have to read descriptions involving several sentences each containing a relational expression such as ‘to the left of’, ‘to the right of’ or ‘in front of’. Then, you will have to draw a schema representing each description. It is of course not necessary to draw the items mentioned in the description, writing their name is sufficient. If you think that several schemas are compatible with a description, draw only the first that comes to your mind”. There was no specific restriction regarding the time when participants could draw the schema. Hence, participants could start their schema after reading the first or the last sentence of the description. Participants were tested individually and acted as their own control.

Results and discussion

In what follows, we first describe the kinds of schemas participants used, we then characterize the data in general terms before focusing on the specific preferences among participants. We distinguished four types of schemas: (1) Schemas in line with the “manipulation difficulty” hypothesis; for these schemas the third item, C, was placed outside the relation formed by the first two items, A and B; (2) schemas in line with the pragmatic hypothesis; for these schemas the C item was inserted between A and B; (3) schemas that did express the indeterminacy of the first two sentences despite the fact that participants were required not to do so, and finally; (4) genuine erroneous schemas, that is schemas that were not compatible with the given description. Table 2 presents the percentages of these four types of schemas for the eight indeterminate descriptions.

Participants exhibited a clear preference for “Outside” schemas over “Inside” ones (61 vs. 25%). The mean difference between “Outside” and “Inside” answers across

Table 2 The percentages of schemas for each description

	Description Type								Mean
	A left B A left C	A left B C right A	A right B A right C	A right B C left A	B left A A right C	B left A C left A	B right A A left C	B right A C right A	
Outside	70.6	47.1	64.7	47.1	52.9	82.4	47.1	76.5	61.0
Inside	17.6	35.3	23.5	29.4	35.3	11.8	35.3	11.8	25.0
Indeterminate	5.9	5.9	11.8	17.6	5.9	5.9	0.0	11.8	8.1
Erroneous	5.9	11.8	0.0	5.9	5.9	0.0	17.6	0.0	5.9
Total	100	100	100	100	100	100	100	100	100

all participants was 2.88 (SD = 5.04), a value that significantly differs from 0 ($t(16) = 2.36, P < 0.05$). Among the 17 participants, 13 drew more “Outside” schemas than “Inside” ones and four drew more “Inside” schemas than “Outside” ones (Binomial test, $P < 0.05$, two-tailed). All descriptions elicited more “Outside” schemas than “Inside” ones.

We also compared descriptions for which C was the subject of the second sentence to descriptions for which C was the predicate and we observed no difference (63.2 vs. 58.8% of “Outside” schemas, Wilcoxon signed rank test $N = 17, T = 17, z = 0.65$, NS, and 22.1 vs. 27.9% of “Inside” schemas, Wilcoxon signed rank test, $N = 17, T = 27.5, z = 0.49$, NS). Finally, we observed that participants produced more “Outside” schemas for homogeneous descriptions than for heterogeneous ones (73.5 vs. 48.5%, Wilcoxon signed rank test, $N = 17, T = 4, z = 2.75, P < 0.01$) and consequently more “Inside” schemas for heterogeneous descriptions than for homogeneous ones (33.8 vs. 16.2%, Wilcoxon signed rank test, $N = 17, T = 3, z = 2.31, P < 0.05$). This result was not expected by a priori hypothesis and was observed neither in Experiment 1 nor in subsequent experiments.

In contrast to the previous experiment, Experiment 2 uses a methodology that elicits a clear preference for one representation type over the other. The data provide clear support for the “manipulation difficulty” hypothesis and questions the possibility of observing the pragmatic effect hypothesised above. However, two remarks need to be made regarding this issue. First, according to the pragmatic analysis the speaker is providing an online description of what she is looking at. This implies that the situation described in the first premise happens before the situation described in the second premise. We may presume that this temporal difference was not made explicit in the present experiment. There is actually no direct cue indicating that the description is an online account of what the speaker is currently observing. The sentence order may be thus seem quite arbitrary. One way to increase the rate of schemas in line with the pragmatic hypothesis would thus be to highlight the fact that the event described in the second premise occurs unambiguously after the event described in the first premise. The next experiment will aim to investigate this issue.

A second issue concerns the methodology employed in Experiment 2. One may assume that it introduces a bias in favour of the manipulation difficulty hypothesis by rendering the construction of the “Inside” representation difficult. Indeed, the very fact that participants have to write down the configuration of the items conveyed by the description, may prevent them from manipulating the items in a way that conformed to the pragmatic hypothesis. This is because once the items are represented they cannot be

moved. Consider for instance the two following indeterminate statements:

The banana is to the left of the apple
The strawberry is to the left of the apple

If, as one can reasonably assume, the participant starts by representing the first statement, she will come up with this initial schema:

Banana	Apple
--------	-------

She will then have trouble putting the word “strawberry” between the banana and the apple simply because it is difficult to move the Banana on the left (or the apple on the right): She would need to erase the word Banana to leave enough space for the word Strawberry. Of course after one or two indeterminate descriptions, the participant may anticipate that and leave enough space between the first two items so that a third one could be inserted between them, but the fact remains that our methodology is susceptible of favouring the “manipulation difficulty” hypothesis. The fourth experiment will address this issue and will aim to manipulate the ease with which the third item can be inserted between the first two. Finally, asking participants to represent the first schema that came to their mind might also have encouraged them to search for the most economical schema rather than to search for the best representation. Hence, in the next two experiments, participants were required to construct the best representation when several were possible.

Experiment 3

The purpose of this experiment was to compare a condition that emphasises the temporal difference between the first two sentences (experimental condition) to a condition that does not (control condition). We devised an experimental condition in which the first sentence could be processed as a reference frame and the second one as a new piece of information that modified that frame. We thus expected such a condition to result in more “Inside” schemas than the control one.

Method

Participants

Two hundred and thirty seven undergraduate students of psychology and history from the University Louis Lumière

(Lyon) and from the Catholic University of Lyon took part in this experiment.

Material and design

In this experiment 2 participants received a single indeterminate description that contained only two relational sentences involving three items. The description concerned a painting on which three fruits were depicted. In the control condition the two relations were introduced in the same way whereas in the experimental condition, the second relation was said to have been drawn by a painter after the first relation was initially given:

Control condition:	Experimental condition:
On John's painting,	On John's painting,
B is to the left of A.	B is to the left of A.
C is to the left of A.	Then, John puts C to the left of A.

To make the second sentence more natural we added an agent (namely John) without which the second sentence would have sounded slightly odd in French. Indeed, in the alternative involving no explicit agent (see below), one will sense that the introduction of temporality renders the involvement of an agent moving the fruit implicit. It thus seemed better to acknowledge his existence by naming him.

No agent alternative:

On John's painting,
B is to the left of A.
Then, C is to the left of A.

Only four of the eight possible indeterminate descriptions were used. The four descriptions were those for which the new item C was introduced as the first item of the second sentence. Had it been one of the two items introduced in the first sentence it would have meant that an item in a previously fixed position would now be moved, a situation which is rather unnatural in the context of the experimental condition. Indeed, it is more natural that what "John puts" is a new fruit rather than a fruit already mentioned in the first sentence. Moreover, we chose to use single problems in order to avoid a routinisation procedure that could lead the reader to skip the phrase "Then, John puts" and to focus only on relations. One hundred and twenty participants received the description of the control condition and 117 received that of the experimental condition.

Procedure

As for Experiment 1, participants were asked to draw only one schema even if they thought that several possibilities were compatible with the description. More specifically the instructions read as follows: "In this experiment, you will first have to read a description involving two sentences each containing a relational expression such as 'to the left of or 'to the right of'. The description concerns fruits and vegetables on a painting made by a student in a visual art class. Then, you will have to draw the schema corresponding to the description. If you think that several schemas are compatible with a description, draw only the one that best represents the description". Participants were presented with a horizontal line on which they had to draw their schema. An example with a three-item determinate description was given. Participants were tested in groups of 20–50 individuals.

Results and discussion

Table 3 presents the percentages of the three categories of schemas ("Outside" schema, "Inside" schema, and erroneous/indeterminate schema) in the two conditions. As in Experiment 1, participants in the control condition drew more slightly "Outside" schemas than "Inside" ones (54.2 vs. 45.8% when indeterminate and erroneous schemas are discarded) but the difference is not significant (Binomial test, not significant). In contrast, the experimental manipulation resulted in a clear-cut effect since the percentages of schemas highly differ across conditions ($\chi^2(2) = 12.9$; $P < 0.005$). To determine where the differences between conditions lie, the chi-square was partitioned into degrees of freedom (Castellan, 1965). In line with the pragmatic analysis, the experimental condition yielded more "Inside" schemas than the control condition ($\chi^2(1) = 11.1$; $P < 0.001$) and the proportion of erroneous/indeterminate

Table 3 The percentages of schemas in the control and experimental conditions

	Control condition ($N = 120$)	Experimental condition (<i>Then, John puts</i>) ($N = 117$)
"Outside" schemas	48.3	29.9
"Inside" schemas	40.8	64.1
Erroneous and indeterminate schemas	10.9	6
Total	100	100

answers did not differ across conditions ($\chi^2(1) = 1.8$; NS). Interestingly, participants of the experimental condition drew significantly more “Inside” schemas than “Outside” ones (68.2 vs. 31.8% when indeterminate and erroneous schemas are discarded, Binomial test, $P < 10^{-3}$). Finally, the proportion of “Outside” schemas did not significantly differ between homogeneous and heterogeneous descriptions (43.8 vs. 34.5%, $\chi^2(1) = 2.16$; NS).

The experimental manipulation had a clear impact on the way participants interpreted the second sentence in the context of the first one. This experiment shows that when one emphasizes the fact that the event described by the second sentence occurs after the event described by the first one, people are more likely to interpret relational expressions like “to the left of” or “to the right of” as “immediately to the left of” or “immediately to the right of”. While the pragmatic interpretation is not the most favoured in the control condition, it becomes favoured when the context introduces a temporal difference between the events described by each sentence. In contrast to Experiment 2, the control condition did not yield a rate of schemas following minimal manipulation that was significantly higher than the rate of schemas following the pragmatic hypothesis, though the trend was in the same direction. This may result from the instructions that did not ask to draw the first diagram that came to mind as in Experiment 2 but that asked to draw the diagram that best represented the description. Participants may have interpreted the instruction of Experiment 2 as a request to draw the easiest (i.e. the one involving the littlest amount of manipulation) diagram while in Experiment 3 a greater part of participants may have interpreted the instructions as a request to draw the schema that best captured what the speaker meant by the description. In Experiment 4, one experimental condition was similar to the control condition of Experiment 3 (the only difference lay in the fact that participants of Experiment 4 were individually tested). We will thus see whether the absence of difference in the paper and pencil condition persists or not. However, this was of course not the main purpose of Experiment 4.

Experiment 4

The purpose of this experiment was to test whether the pragmatic representation would occur more often when it becomes easier to insert the third item between the first two. As indicated above, representing the configuration conveyed by the description with paper and pencil is problematic since once the first two items are represented there might not be enough room to insert the name of the third item between the names of the first two. The paper and pencil methodology may thus exaggerate the difficulty

of building the pragmatic representation. We thus manipulated the ease with which such a representation could be achieved. If participants are guided by pragmatic considerations then they should be more likely to choose the “Inside” representation when it becomes easier to construct it and one can also predict that such a representation will predominate if it is not harder to construct than the “Outside” representation.

This experiment involves three conditions that will be detailed in the Method section. In the “paper and pencil” condition, building the “Inside” representation was relatively hard. In the “real fruits” condition, the first two items could be easily manipulated in order to leave enough room to insert a third item. In the “options” condition, the additional effort needed to put the third item between the first two was entirely removed; it was therefore no harder to insert the third item between the first two than to put it outside. The predictions that result from the pragmatic analysis are thus as follows: (1) The “options” condition will result in more “Inside” representations than the “real fruits” condition; (2) The “real fruits” condition will result in more “Inside” representations than the “paper and pencil” condition; (3) The “options” condition will result in more “inside” representations than “outside” ones.

Method

Participants

Two hundred and seventy three students from the University Louis Lumière (Lyon) took part in this experiment. There were 96 participants in the “paper and pencil” condition, 90 in the “real fruits” condition and 87 in the “options” condition. They were all native speakers of French.

Design and material

As for Experiment 3, participants received a single indeterminate description that involved two relational sentences and three items. The descriptions used here were the same as those of the control condition of Experiment 3 and also concerned a painting on which three fruits were drawn. This experiment included three conditions that presented the descriptions in an identical manner. The three conditions only differed in the means participants could use to represent what was conveyed in the description. As for Experiments 2 and 3, in the “paper and pencil” condition, participants had to use paper and pencil. In the “real fruits” condition, participants were required to depict the spatial description with real fruits

provided by the experimenter (namely a peach, an orange and an apple). In this condition, the items described in the two statements can thus be easily manipulated. Finally, in the “options” condition, participants were given paper and pencil but the relation described by the first sentence was already represented by a schema that offered three positions for the third item (see examples below). Two schemas that differed according to the position available were chosen. For one schema the three positions were symmetrical: One position on the left of the first two items, one on the right and one in between; and for the other schema they were asymmetrical: two positions on the left/right of the first two items and one in between. The use of those different schemas enabled us to assess a potential bias induced by the symmetrical schema. Indeed for such a schema, the very fact that the “inside” position is in the middle of the line could have encouraged participants to choose that position. Here are the two possible schemas for the statements “the peach is to the left of the orange; the apple is to the left of the orange”:

Symmetrical schema:

_____ peach _____ orange _____

Asymmetrical schema:

_____ _____ peach _____ orange

With such schemas it is not more difficult to insert the apple between the peach and the orange than to put it on the left of these two (the right position being the wrong one).

Procedure

In the three conditions, participants were individually tested in the open-air lobby of the University and were seated at a table with the experimenter standing near them. In the “real fruits” condition, the experimenter took note of the representation worked out by the participants. For the three conditions, participants were instructed to recreate the disposition of the objects in the description. An example involving a three-term determinate description was given and was represented on a horizontal line. In the “real fruits” condition, the example was represented by a photograph of three real fruits whereas in the other two conditions it was represented by a schema involving the names of the fruits. For the three conditions, participants were informed that they had to choose the configuration that best represented the description if they thought that several configurations were possible.

Results and discussion

Table 4 presents the percentages of three categories of representation (“Outside” representation, “Inside” representation and erroneous representation) in the three conditions. We first verify that as in Experiment 2, the “paper and pencil” condition elicited significantly more “Outside” representations than “Inside” ones (73.8 vs. 26.2% when erroneous representations are discarded, Binomial test, $P < 10^{-4}$, two-tailed) and we also note that for the “real fruits” condition this difference is not significant (53% vs. Binomial test NS).

Second, we present our results by examining each of our three predictions and we test whether the three conditions differed with respect to the type of representation by means of a chi-square test. In order to meet Cochran’s recommendation (Cochran, 1954, cited by Siegel & Castellan, 1988) that no more than 20% of the cells should have an expected frequency lesser than 5 when the degree of freedom of a chi-square test is greater than 1, we discarded participants that built erroneous and indeterminate representations and focused only on participants who produced either an “Inside” representation or an “Outside” representation. The chi-square test revealed that the three conditions strongly differ with respect to the type of representation ($\chi^2(2) = 69.8; P < 10^{-18}$). To determine where the differences between conditions lie, the chi-square was partitioned into degrees of freedom. In line with Prediction 2, the “real fruits” condition resulted in more “Inside” representations than the “paper and pencil” condition ($\chi^2(1) = 7.33; P < 0.01$) and those two conditions combined yielded fewer “Inside” representations than the “options” condition ($\chi^2(1) = 62.47; P < 10^{-14}$). Prediction 1 was also confirmed: the “options” condition yielded significantly more “Inside” representations than the “real fruits” condition ($\chi^2(1) = 35.15; P < 10^{-8}$). The “options” condition elicited more “Inside” than “Outside” representations (Binomial test, $P < 10^{-12}$, two-tailed), a

Table 4 The percentages of schemas in the three conditions: “paper and pencil”, “real fruits” and “options”

	Paper and pencil (N = 96)	Real fruits (N = 90)	Options (N = 87)
“Outside” representation	64.6	53.3	11.5
“Inside” representation	22.9	46.7	88.5
Erroneous and indeterminate representations	12.5	0	0
Total	100	100	100

result that corroborates Prediction 3. Let us also note that in the “options” condition, the rate of “Inside” representations did not differ with respect to the two types of positions provided (89.7% for symmetrical positions vs. 87.5% for asymmetrical positions, $\chi^2(1) = 0.10$; NS). Finally, a chi-square test revealed that homogeneous and heterogeneous descriptions did not significantly differ with respect to representation type ($\chi^2(2) = 1.27$; NS).

This experiment provides insight into the way pragmatics and manipulation difficulty affect representation. When the pragmatic representation is hard to work out, the alternative representation, involving minimal manipulation, is favoured. However, when both types of representation are equally accessible, the pragmatic representation prevails, as it is shown by the “options” condition. If participants had only been guided by considerations of manipulation difficulty, the two types of representation should have been produced at similar rates in this condition.

General discussion

This study aimed to identify the factors that guide the representation procedure of indeterminate spatial relations. We proposed that participants develop a representation that minimises effort and that therefore reduces the manipulation of entities referring to the items mentioned in the description. Pragmatic considerations lead to the assumption that participants should develop a representation that reflects the fact that the description is given by a speaker who communicates appropriately. In certain situations, these two factors can favour different interpretations of the same utterance, i.e. “C is to the left of A” in the context of the preceding sentence “B is to the left of A”. According to the view that people try to minimise manipulation effort, C will be represented to the left of A as well as *to the left of B* while according to the pragmatic perspective, C will be represented to the left of A but *to the right of B* (i.e. C will be immediately to the left of A). A greater incidence of one representation over the other should reveal the extent to which one factor predominates over the other. This is what we wanted to investigate in the experiments reported here.

Although the first experiment did not exhibit any predominance of one representation type, the three following experiments succeeded in showing a preference for one type over the other. These experiments show that both the pragmatic factor and the manipulation difficulty factor act on the way people deal with indeterminacy. On the one hand, when the representation mode is rigid, that is when the pragmatic representation becomes costly, the alternative representation, requiring minimal manipulation, predominates as shown by Experiments 2 and 4 (“paper

and pencil” condition). On the other hand, the pragmatic representation prevails when (1) it does not involve more effort than the representation following minimal manipulation (the “options” condition of Experiment 4) and when (2) pragmatic cues are included in the description (the experimental condition of Experiment 3).

Of course, the effects we observed in the last three experiments occur in conditions where participants externalised the representation of the description they received and Experiment 4 shows that different modes of externalisation (paper and pencil vs. real objects) lead to different patterns of results. One may thus wonder whether an externalised representation and an internalised representation result in different ways of dealing with indeterminacy and one may also wonder which of the various modes of externalisation is more similar to internalisation. While some researchers highlight the similarities between external and internal spatial representations (Larkin and Simon, 1987; Huttenlocher, 1968; Schaeken, Van der Henst, & Schroyens, 2007), others argue that they are likely to induce different strategies (Newton & Roberts, 2000; Roberts, Gilmore, & Wood, 1997).

How could internalisation and externalisation prompt the construction of one model over the other? We envisage two options. On the one hand, the internalised representation is cognitively more demanding than the externalised representation because it requires maintaining all the information conveyed by the description in working memory. This constraint may reduce the possibility of manipulating the items within the model and favour the construction of the model which requires less modification of its initial part (i.e. the “Outside” model), and thus less effort, than the “Inside” model (see introduction and Fig. 1). On the other hand, the manipulation and the modification of the representation might be made more easily when it is internal than when it is based on paper and pencil. As already said in the discussion of Experiment 2, once two items are externally represented with paper and pencil they cannot be moved. There is no such a constraint for internalised representation where items can be mentally moved (as for instance shown by mental rotation tasks). According to this second option, an externalisation procedure involving paper and pencil will give rise to more representations based on minimal manipulation than an internalisation procedure.

To disentangle these two possibilities the comparison of the first two experiments provides preliminary cues because (1) both experiments use the same descriptions and (2) Experiment 1 prompts internalisation whereas Experiment 2 prompts externalisation with paper and pencil. The data indicate that internalisation in Experiment 1 produces proportionally fewer “Outside” models (granted that an incorrect answer genuinely reveals the

construction of a mental model rather than a random response) than externalisation with paper and pencil in Experiment 2 (61 vs. 27% Mann–Whitney test, $z = 2.86$, $P < 0.005$). Moreover, among the different externalisation procedures used in this study, the one that seems the most comparable to internalisation is the one that uses real objects (see Experiment 4) since in both cases the items can be moved either mentally or physically. Interestingly, the sets of data obtained both in Experiment 1 (internalisation) and in the “real fruits” condition of Experiment 4 (externalisation) showed no preference for the “Outside” or the “Inside” representation. This seems to indicate that internalisation procedure and externalisation procedure with real objects give rise to similar results (see also Schaeken, Van der Henst, & Schroyens, 2007). However, additional data would need to be collected to compare internalisation and various forms of externalisation in more systematic ways.

The fact that one can easily manipulate how people will represent two indeterminate premises is quite instructive about the way they interpret spatial prepositions. It indicates that the interpretation of “to the left of” and “to the right of” is quite flexible. At a strict semantic level, an expression such as “to the left of” means “anywhere to the left of”. As shown by the data, participants did not have any difficulty in considering that C was to the left of A even if an item B was between C and A. Similarly, one would probably accept that, in the following configuration the item C is to the left of B although it departs from the horizontal line set up by the B–A pair:

C
B A

Hence, at the semantic level, the position of C described by the “C is to the left of X” is not fully determined. This basic and relatively underdetermined level of interpretation can however be enriched if the audience assumes that the speaker tries to provide the best description. By the very fact that the speaker aims to be helpful when she claims that “C is to the left of A”, the audience can infer that C is not anywhere to the left of A, and in particular that there is no item between C and A, because if it would have been the case the speaker would presumably have chosen another way to describe the position of C. In this context, saying “C is to the left of A” narrows the meaning of “left of” so that the listener understands the utterance to indicate “C is immediately to the left of A”.

These two levels of analysis concerning spatial expressions parallel those made for other expressions such as propositional connectives (*and, or, if... then*) or quantifiers (*some*). Such terms elicit pragmatic inferences that enrich their semantic and relatively weak meaning. These

inferences are often described under the label of “conversational implicatures”, a concept introduced by Grice (1975; see also Horn, 1972, 1989; Levinson, 1983). Generally speaking, a conversational implicature is a pragmatic inference that is meant by a speaker but that goes beyond the explicit linguistically encoded meaning of what is said. In other words, it is not a part of what is said, but it can be inferred from what is said. Take the example of the term *some*. When one says *Some P are Q*, one often implies that *Some, but not all P are Q* (e.g. “Some of my friends are married” implies that “Some, but not all of my friends are married”). This interpretation goes beyond and even contradicts the semantic or logical characterisation of *some* which is compatible with *all* (If *all P are Q*, it necessarily follows that *some P are Q*). In Gricean terms, the fact that the speaker is cooperative and should make a contribution as informative as is required (Grice’s first maxim of quantity), implies that if the speaker is in a position to use the stronger and more informative term *all*, she should do so. If she does not, this means that the speaker is not in position to use it and hence the implicature *not-all* is likely to follow from the saying of the term *some*. In the case of spatial relations, if a cooperative speaker chooses to utter “A is to the left of C” in a configuration where B is already to the left of C, the hearer can infer that she is not in a position to utter that A is to the left of B. Had it been the case she should have uttered “A is to the left of B” as it is more informative (i.e. less ambiguous). This would lead the hearer to build the B–A–C model rather than the A–B–C model.

Some of the data presented here also echo recent experimental work on pragmatic inferences. In the present study, when the effort required to work out the pragmatic interpretation was relatively high it tended to disappear, showing thus that it was not automatically and necessarily made. Similarly, in an experimental study aiming to investigate the time course of the pragmatic inference linked to the quantifier *some*, Bott and Noveck (2004, Experiment 4), observed that the *some but not all* interpretation (i.e. the pragmatic interpretation) was less likely to occur when cognitive resources were rendered limited. The availability of cognitive resources was manipulated with the time given to participants to judge whether categorical statements such as *some elephants have trunks* were true or false. Participants who had a relatively short time (900 ms) to evaluate the categorical statement, yielded more logical responses (answering TRUE to *some elephants have trunks*) than participants who had a relatively long time (3,000 ms; 72 vs. 56%). Bott and Noveck interpreted this finding as a support for the view that the *not-all* pragmatic inference is effortful and is not automatically triggered.

Similarly, the fact that the pragmatic model is not always chosen indicates that the “immediately-to-the-left” interpretation is not automatically triggered. Indeed, our experiments indicate that context influences the interpretation of spatial terms and thus the way people build mental models. It is important that these contextual factors are well understood and controlled in tasks involving mental model construction.

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